Combined Trellis and Iterative Decoding of GMSK and FQPSK-B

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INTRODUCTION

Due to already congested space frequencies and growing demand for high data rates, modulations with bandwidth efficiency and sharp spectral roll-off are becoming increasingly important to avoid interference with adjacent channels. Two such modulations recommended by the Consultative Committee for Space Data Systems (CCSDS; see Rec. 2.4.17A) with constant or near-constant envelope properties suitable for space applications are GMSK (Gaussian Minimum Shift Keying) and FQPSK-B (Feher-patented QPSK). However, as with most bandwidth efficient modulations, GMSK and FQPSK-B suffer degradation in terms of bit error rate (BER) for a given E_b/N_o with respect to non-bandlimited modulations such as unfiltered BPSK.

In an effort to find optimal receivers for GMSK and FQPSK-B, trellis coded interpretations of the two modulations were described in [5] and [7], respectively. In addition, error correcting codes such as the industry standard rate 1/2, k=7 convolutional code are often applied to compensate for the degradation, albeit at the expense of bandwidth. In this paper, we consider two different methods of decoding convolutionally coded GMSK and FQPSK-B using iterative detection and combined trellis decoding.

COMBINED TRELLIS DECODING

Fig. 1 shows the FQPSK-B encoder with the standard rate 1/2, k=7 convolutional code. The FQPSK encoder has 16 states with 4 transitions to each state and 6 outputs which are used to map the I and Q channel outputs to one of sixteen different waveforms [7]. A interleaver is placed between the convolutional code and the FQPSK-B encoder to break up error bursts at the output of the FQPSK-B Viterbi decoder. The trellis demodulator for FQPSK-B consists of a bank of sixteen correlators matched to the waveforms and a Viterbi algorithm (VA) to find the maximum likelihood sequence through the FQPSK-B trellis. For more details on the FQPSK-B Viterbi receiver, see [7].

The conventional approach in the case of a convolutional code concatenated with a TCM would be to decode the two codes separately; i.e., taking the soft outputs from the FQPSK-B Viterbi receiver, de-interleaving them, and using them as inputs to the convolutional code Viterbi algorithm. However, this is an suboptimal solution since soft information is lost from the inner decoder to the outer decoder due to quantization and the fact that the Soft Output Viterbi Algorithm (SOVA) [3] output is only an approximation to the true log-likelihood ratio. Instead, we propose merging the FQPSK-B and convolutional code trellises together into a combined trellis. Since there is now only one Viterbi algorithm, no interleaver is needed and no soft information is lost between decoders. Removing the interleaver reduces latency and memory requirements, and the combined trellis decoding provides improved detection performance.

The tradeoff for combined trellis decoding is the additional computation complexity in the VA. In general when two code trellises are combined, decoding complexity increases exponentially. For example, if the inner binary code has m states and the outer binary code has n states, the combined trellis may have up to mn states compared with a just total of $m+n$ states if the trellises are treated separately. However, for quadrature trellis coded modulations with short constraint lengths such as GMSK and FQPSK-B, the increase in complexity is not so large. In addition, there are simplifications which can make combined trellis decoding only slightly more complex than separate decoding of the codes while still providing improved bit error performance.

From Fig. 1, it can be noticed that the states of the I and Q FQPSK encoders are simply delayed outputs of the rate 1/2 convolutional encoder. Thus knowledge of the previous two states of the rate 1/2 encoder determines exactly the current state of the FQPSK encoder. The entire FQPSK encoder can be replaced by adding two more delay units to

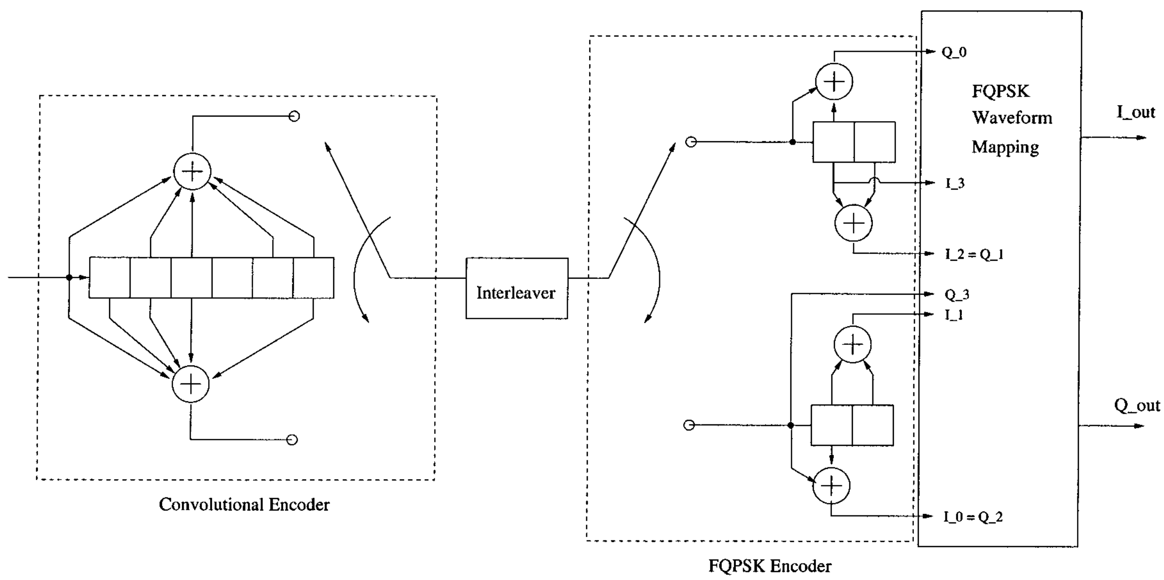

Figure 1: FQPSK encoder with Convolutional Code

the convolutional code shift register and adding the appropriate taps. It can be shown that six FQPSK-B outputs in the combined encoder are:

$$
\begin{aligned}
Q_3 &= a_k \oplus a_{k-1} \oplus a_{k-2} \oplus a_{k-3} \oplus a_{k-6} \\
Q_2 &= I_0 = a_k \oplus a_{k-4} \oplus a_{k-6} \oplus a_{k-7} \\
Q_0 &= a_k \oplus a_{k-1} \oplus a_{k-2} \oplus a_{k-4} \oplus a_{k-5} \oplus a_{k-7} \\
I_3 &= a_{k-1} \oplus a_{k-3} \oplus a_{k-4} \oplus a_{k-6} \oplus a_{k-7} \\
I_2 &= Q_1 = a_{k-1} \oplus a_{k-2} \oplus a_{k-3} \oplus a_{k-5} \oplus a_{k-6} \oplus a_{k-8} \\
I_1 &= a_{k-1} \oplus a_{k-5} \oplus a_{k-7} \oplus a_{k-8}
\end{aligned} \tag{1}
$$

where a_k is the current input bit to the convolutional encoder, a_{k-1} is the previous input bit, etc. Thus the six octal generators for the Q_3, Q_2, Q_0, I_3, I_2, I_1 outputs of the combined encoder are $\{744, 426, 355, 732, 262, 213\}$.

The combined FQPSK-B trellis encoder is shown in Fig. 2. The combined FQPSK-B trellis has 256 states with 2 transitions per state for a total of 512 branch metric computations per decoded bit. For comparison with the convention decoding of concatenated codes, we use the approximation that the SOVA has about double the complexity of a VA (depending on the actual implementations of the algorithm, the SOVA is about 1.5 to 2 times the complexity of the VA). Thus the FQPSK SOVA requires roughly the equivalent of 128 branch metric computations per decoded bit (2 x 16 states x 4 transitions per state) and the convolution VA also requires 128 branch metric computations per decoded bit. Thus, decoding the two trellises separately requires a total of 256 branch metric computations per decoded bit. The combined trellis requires roughly double this number of branch metric computations but has only one encoder structure as opposed to two and does not have the latency and additional memory requirements associated with the interleaver/deinterleaver.

Simulation Results for Combined Trellis Decoding of Coded FQPSK-B

The bit error rate performance of rate 1/2, k=7 convolutionally coded FQPSK-B using the combined trellis VA decoder was simulated using Signal Processing WorkSystem (SPW) software. The VA of the combined trellis used a truncation path length of 50 bits. The simulated channel consisted of additive gaussian noise and a saturated 10 Watt ESA SSPA model whose AM/AM and AM/PM characteristics can be found in [6]. The simulations were performed in floating point.

Fig. 3 shows the simulated bit error performance of the combined trellis FQPSK-B decoder. For comparison, the bit error rate of FQPSK-B with the same convolutional code but using symbol-by-symbol detection and a Viterbi decoder is shown. As the figure shows, the BER using the combined trellis VA is 0.4 dB E_b/N_o better than symbol-by-symbol detection of

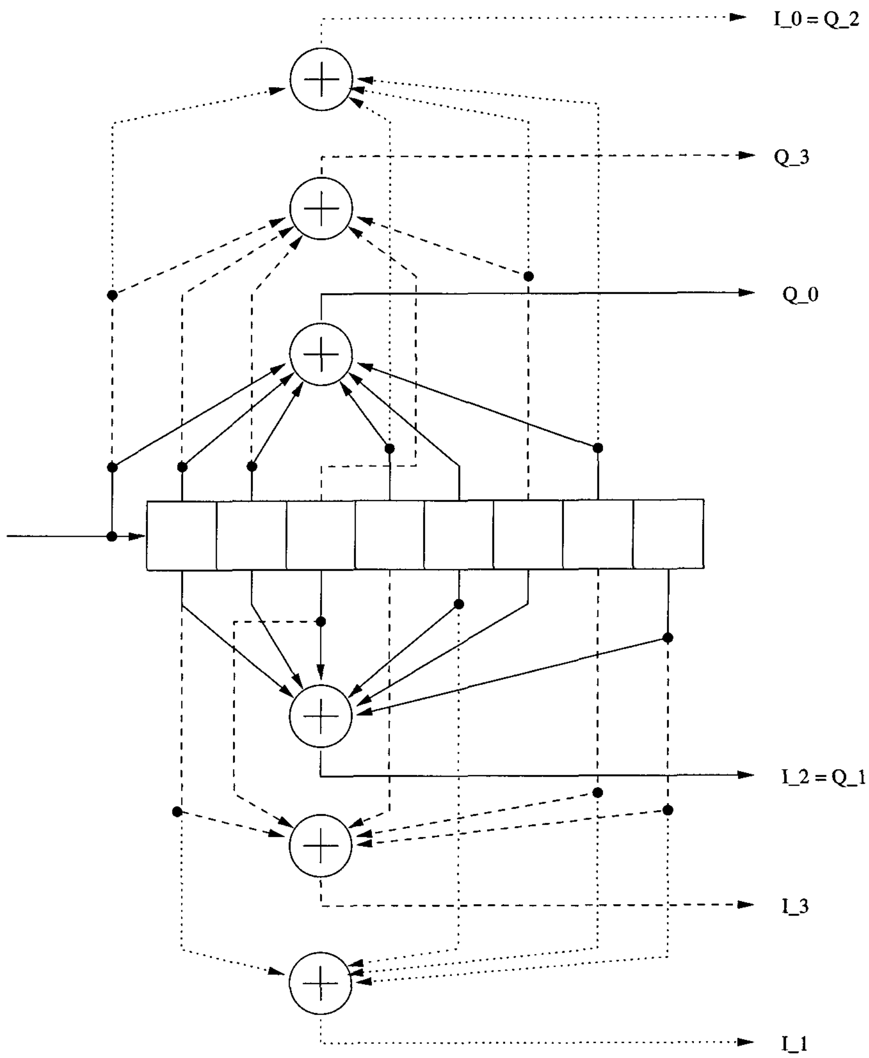

Figure 2: FQPSK combined trellis encoder

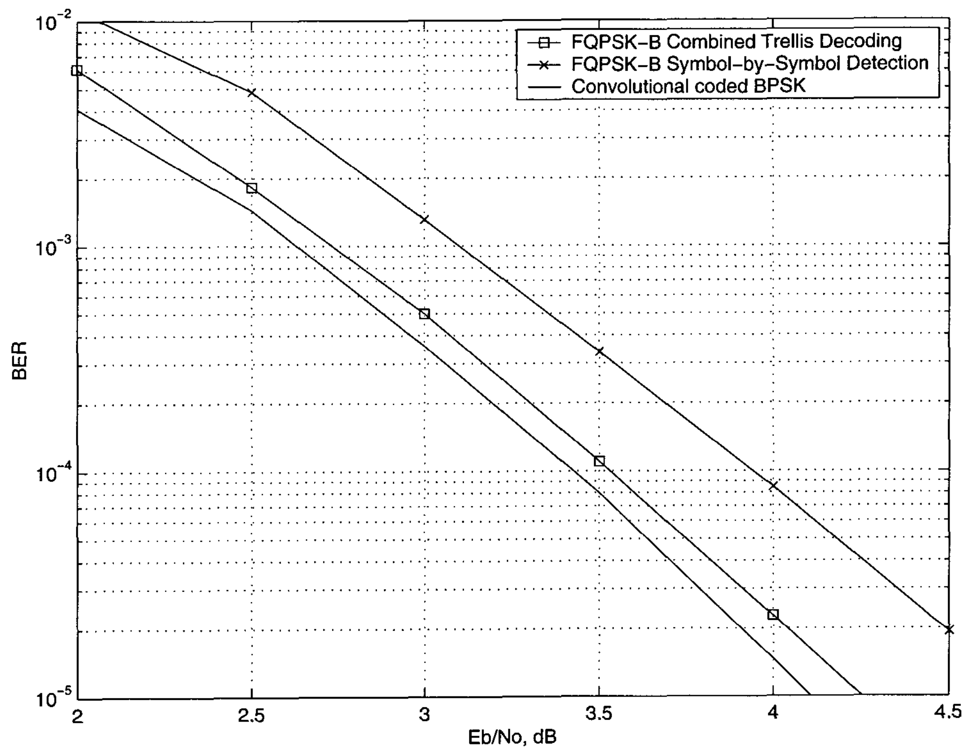

Figure 3: Bit error performance of FQPSK combined trellis VA

FQPSK-B and a separate VA, and within 0.1 dB of BPSK using the same code.

GMSK Combined Trellis Decoding

Using the Laurent decomposition of GMSK [4], GMSK can be interpreted as an amplitude modulated I-Q trellis coded modulation. In this study, we will consider GMSK $BT_b = 0.25$ which has a spectrum comparable to FQPSK-B and has also been recommended by the CCSDS. For the Laurent decomposition of GMSK $BT_b = 0.25$, good performance can be achieved by only considering the two largest pulses $h_0(t)$ and $h_1(t)$. The resulting GMSK trellis has 4 states and complex outputs $a_{0,n}$ and $a_{1,n}$ which are used to amplitude modulate $h_0(t)$ and $h_1(t)$, respectively [5]. In precoded form [5], the coefficients for $a_{0,n}$ and $a_{1,n}$ are given by:

$$\begin{aligned}
Re\{a_{0,n}\} &= \begin{cases} d_n & \text{n even} \\ 0 & \text{n odd} \end{cases} \\
Im\{a_{0,n}\} &= \begin{cases} 0 & \text{n even} \\ d_n & \text{n odd} \end{cases} \\
Re\{a_{1,n}\} &= \begin{cases} 0 & \text{n even} \\ d_n d_{n-1} d_{n-2} & \text{n odd} \end{cases} \\
Im\{a_{1,n}\} &= \begin{cases} d_n d_{n-1} d_{n-2} & \text{n even} \\ 0 & \text{n odd} \end{cases}
\end{aligned} \tag{2}$$

where d_n are the binary input bits. The encoding of $a_{1,n}$ in 2 would suggest a single shift register of two bits. However, by dividing the data into I and Q streams, this serial shift register for GMSK can be split into two parallel shift registers of only 1 bit each. Thus, when combined with the standard rate 1/2, k=7 convolutional code, the combined GMSK trellis has only 128 states with two transitions per state.

Fig. 4 shows the combined trellis encoder after simplification. The octal generators for $Re\{a_{0,n}\}$, $Re\{a_{1,n}\}$, $Im\{a_{0,n}\}$ and $Im\{a_{1,n}\}$, are $\{448, 364, 466, 075\}$ respectively. Like FQPSK-B, the combined trellis GMSK receiver correlates the incoming signal with $h_0(t)$ and $h_1(t)$ and feeds the metrics into the combined trellis VA.

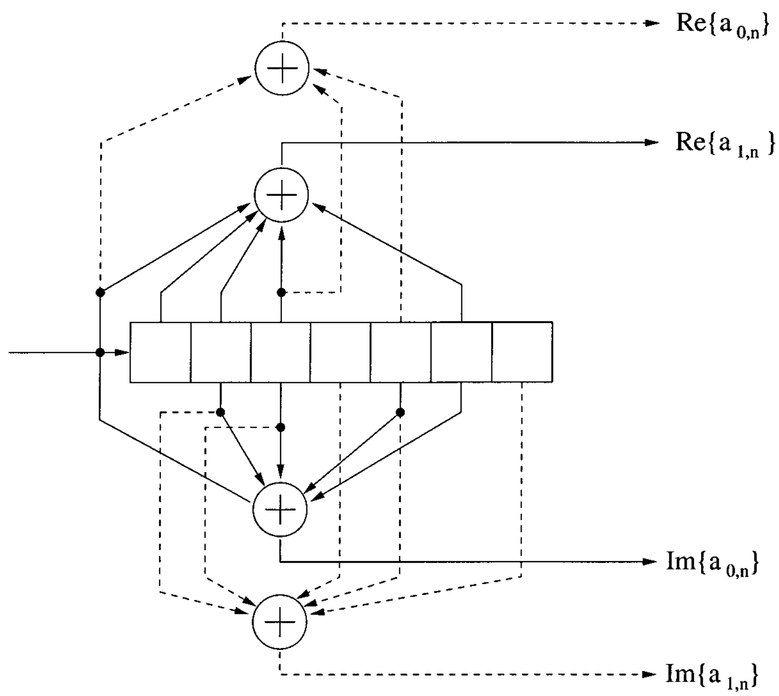

Figure 4: GMSK combined trellis encoder

Simulation Results for Combined Trellis Decoding of Coded GMSK BT_b=0.25

SPW simulations were run for rate 1/2, k=7 convolutionally coded GMSK $BT_b = 0.25$ with combined trellis demodulation. A truncation path length of 50 bits was used. Fig. 5 shows the simulated bit error rates of GMSK with combined trellis decoding. The BER of rate 1/2, k=7 coded BPSK is shown for comparison. With combined trellis demodulation, the implementation loss of GMSK $BT_b = 0.25$ is less than 0.1 dB E_b/N_o with respect to BPSK using the same convolutional code.

ITERATIVE DETECTION

Iterative detection of serial concatenated convolutional codes (SCCC) have been shown to yield remarkable coding gains, and may possess some advantages to parallel concatenated convolutional codes (sometimes called "turbo" codes) [1]. Using the trellis-coded interpretation of GMSK and FQPSK-B, a SCCC can be created by applying a short constraint length convolutional code and pseudo-random interleaver before the TCM.

The FQPSK trellis code can be restructured as a recursive code [8]. This property is important for the inner coder of the SCCC in order to achieve interleaving gain (i.e., increased coding gain with interleaver size). The interleaver in the SCCC serves to enlarge the effective block size of the code, and has random-like properties for best performance. The iterative receiver (see Fig. 6) consists of a bank of filters matched to the modulation waveforms, followed by a soft-in-soft-output (SISO) decoder for the TCM. In our simulations, the SISO is mechanized with a forward-backward algorithm [2]. The extrinsic soft output of the TCM SISO is then deinterleaved and decoded by the convolutional code (CC) SISO. The extrinsic soft output of the CC SISO is then interleaved and sent back as soft input information to the TCM SISO. This decoding loop is iterated over until the soft information has reached a steady state value. It should be noted that in [8], the bit error results for iterative detection of FQPSK using the recursive code is done for the simplified FQPSK trellis (with two 2 states trellises and 2 transitions per state) but not for the full FQPSK trellis as done here. A comparison between the simulation results indicate that iterative detection over the full FQPSK trellis offers a 0.1 dB improve over the simplified trellis for these particular codes.

Using the guidelines set forth in [?], the outer 4-state code was chosen using the following generating polynomial $G(D) = [1 + D + D^2, 1 + D^2]$. This non-recursive code has maximum free distance (which is also odd) for a k=3 rate 1/2 code.

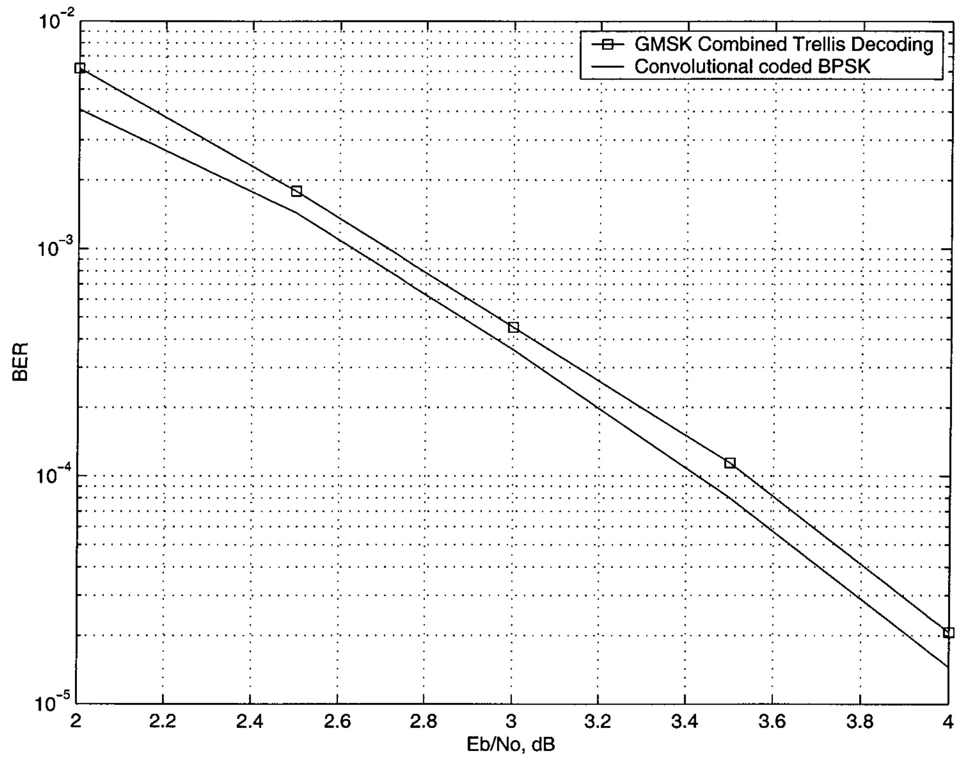

Figure 5: BER of convolutional coded GMSK with combined trellis decoding

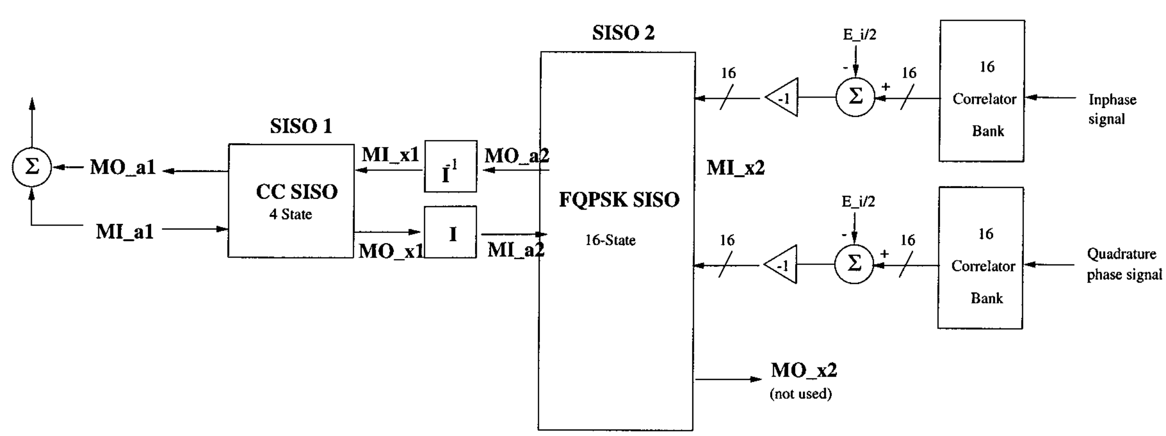

Figure 6: FQPSK-B iterative receiver

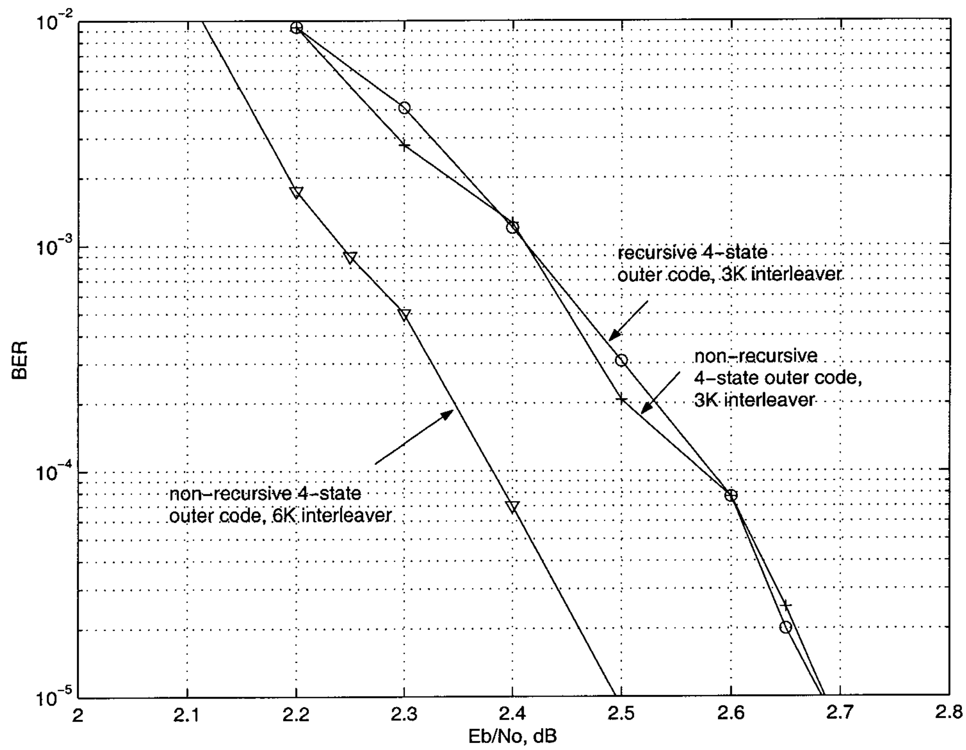

Figure 7: FQPSK-B iterative receiver BER

It is suggested in [1] that non-recursive encoders may be more appropriate as outer encoders since in general they have fewer input errors associated with error events at free distance compared with recursive encoders. For comparison, we also tested the optimal recursive 4-state code used in [8].

Fig. 7 shows the bit error rate of coded FQPSK with iterative detection. Eight iterations were used per decoded block, and a minimum of 30 error blocks were collected per simulation run. The interleaver size used is 3072 bits. As the figure shows, there is virtually no different between the non-recursive code with maximal free distance and the optimal code from [8]. Increasing the interleaver size to 6K provides a 0.2 dB gain over the 3K interleaver.

SUMMARY

Two alternative methods of decoding convolutionally coded GMSK and FQPSK-B are considered using combined trellis demodulation and iterative detection. With combined trellis demodulation, the outer code trellis is combined with the inner modulation trellis to form a single combined trellis. This allows for simplification of the encoder and removal of the interleaver/de-interleaver pair (and the associated latency and memory requirements) while providing improved bit error performance. While combining two such concatenated codes results in exponential usually increases in decoding complexity, the short constraint lengths and I-Q structure of the GMSK and FQPSK trellis codes makes the combined trellis only roughly twice as complex as separate decoding of the codes. With combined trellis decoding, simulation results show that rate 1/2, k=7 convolutionally coded GMSK $BT_b = 0.25$ and FQPSK-B both perform within 0.1 dB E_b/N_o of ideal BPSK with the same code.

Iterative detection treats the convolutional code/trellis coded modulation pair as a SCCC. Using SISO modules and a 3K pseudo-random interleaver, simulations show that FQPSK can reach 10^{-5} BER at 2.7 dB E_b/N_o with a 4-state outer convolutional code and eight iterations. With a 6K interleaver, the receiver E_b/N_o is only 2.5 dB for 10^{-5} BER.

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